Predatory Trading*

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First version: June 15, 2002
This version: February 20, 2003

Abstract

This paper studies *predatory trading*: trading that induces and/or exploits other investors' need to reduce their positions. We show that if one trader needs to sell, others also sell and subsequently buy back the asset. This leads to price overshooting, and a reduced liquidation value for the distressed trader. Hence, the market is illiquid when liquidity is most needed. Further, a trader profits from triggering another trader's crisis, and the crisis can spill over across traders and across assets.

Keywords: Predation, Valuation, Liquidity, Risk Management, Systemic Risk.

*We thank William Allen, Ed Altman, Yakov Amihud, Patrick Bolton, Menachem Brenner, Robert Engle, Stephen Figlewski, Joel Hasbrouck, David Modest, Michael Rashes, José Scheinkman, Bill Silber, Ken Singleton, Jeremy Stein, Marti Subrahmanyam, Peter Sørensen, and Jeff Wurgler, as well as seminar participants at NYU, McGill, Duke University, Carnegie Mellon University, Ohio State University and University of Copenhagen.

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1 Introduction

Large traders fear a forced liquidation, especially if their need to liquidate is known by other traders. For example, hedge funds with (nearing) margin calls may need to liquidate, and this could be known to certain counterparties such as the bank financing the trade; similarly, traders who use portfolio insurance, stop loss orders, or other risk management strategies can be known to liquidate in response to price drops; certain institutions have to (or have an incentive to) liquidate bonds that are downgraded or in default; index funds must re-arrange their portfolios in response to index inclusions and deletions; and intermediaries who take on large derivative positions must hedge them by trading the underlying security. A forced liquidation is often very costly since it is associated with large price impact and low liquidity.

We provide a new framework for studying the strategic interaction between large traders. Some of these traders can be driven into financial difficulty, and their need to liquidate is known by the other strategic traders. All agents trade continuously and limit their trading intensity to minimize the temporary price impact cost.

Our analysis shows that if a distressed large investor is forced to unwind his position (and needs liquidity the most), other strategic traders initially trade in the *same* direction. That is, to profit from price swings, other traders conduct predatory trading and withdraw liquidity instead of providing it. This predatory activity makes liquidation costly and leads to price overshooting. Moreover, predatory trading itself can drive out of the market a vulnerable trader who could have otherwise remained solvent.

These findings are in line with anecdotal evidence as summarized in Table 1. A well-known example of predatory trading is the alleged trading against LTCM's positions in the fall of 1998. Business Week wrote that "if lenders know that a hedge fund needs to sell something quickly, they will sell the same asset – driving the price down even faster. Goldman, Sachs & Co. and other counterparties to LTCM did exactly that in 1998."[1] Also, Cai (2002) finds that "locals" on the CBOE pits exploited knowledge of LTCM's short positions in the treasury bond futures market. Another indication of the fear of predatory trading is evident in the opposition to UBS Warburg's proposal to take over Enron's traders without taking over its trading positions. This proposal was opposed on the grounds that "it would present a 'predatory trading risk' because Enron's traders would effectively know the contents of the trading book."[2] Similarly, many institutional investors are forced by law or their own charter to sell bonds of companies which undergo debt restructuring procedures. Hradsky and Long (1989) documents price overshooting in the bond market after default announcements.

Furthermore, our model shows that an adverse wealth shock to one large trader, coupled with predatory trading, can lead to a price drop that brings other traders in financial difficulty, leading to further predation and so on. This ripple effect can

[1] "The Wrong Way to Regulate Hedge Funds", Business Week, February 26th, 2001, page 90.

[2] AFX News Limited, AFX – Asia, January 18th, 2002.

cause a widespread crisis in the financial sector. Consistently, the testimony of Alan Greenspan in the U.S. House of Representatives on Oct. 1st, 1998 indicates that the Federal Reserve Bank was worried that LTCM's financial difficulties might destabilize the financial system as a whole:

> "...the act of unwinding LTCM's portfolio would not only have a significant distorting impact on market prices but also in the process could produce large losses, or worse, for a number of creditors and counterparties, and for other market participants who were not directly involved with LTCM."

Our model also provides guidance for the valuation of large security positions. We distinguish between three forms of value, with increasing emphasis on the position's liquidity. Specifically, the "paper value" is the current mark-to-market value of a position; the "orderly liquidation value" reflects the revenue one could achieve by secretly liquidating the position; and the "distressed liquidation value" equals the amount which can be raised if one faces predation by other strategic traders, that is, with endogenous market liquidity. The analysis shows that the paper value exceeds the orderly liquidation value, which typically exceeds the distressed liquidation value. Hence, if a large trader estimates "impact costs" based on normal (orderly) market behavior then he may under-estimate his actual cost in case of an acute need to sell because predation makes liquidity time-varying and, in particular, predation reduces liquidity when large traders need it the most. Consistently, Pastor and Stambaugh (2002) and Acharya and Pedersen (2002) find measures of liquidity risk to be priced.

Our analysis also has normative implications for the design of hedge fund disclosure requirements. Large hedge funds that are active in illiquid markets should face less stringent disclosure standards than funds holding liquid assets or funds that are smaller in size. This is consistent with the disclosure guidelines described in the consensus statement by the IAFE Investor Risk Committee (IRC), which consists both of hedge funds and hedge fund investors. IRC (2001) states that "large hedge funds need to limit granularity of reporting to protect themselves against predatory trading against the fund's position." In the same vein, market makers at the London Stock Exchange prefer to delay the reporting of large transactions since it gives them "a chance to reduce a large exposure, rather than alerting the rest of the market and exposing them to predatory trading tactics from others."[3]

Our work is related to several strands of literature. First, our model provides a natural example of "destabilizing speculation" by showing that although strategic traders stabilize prices most of the time, their predatory behavior can destabilize prices in times of financial crisis. This contributes to an old debate, see Friedman (1953), Hart and Kreps (1986), DeLong, Shleifer, Summers, and Waldmann (1990), and references

[3]Financial Times, June 5th, 1990, section I, page 12.

therein. Market impact due to asymmetric information is studied by Kyle (1985), while trading with exogenous market impact is studied theoretically by Cetin, Jarrow, and Protter (2002) and empirically by Chen, Stanzl, and Watanabe (2002), and others, but these papers do not consider the strategic effects of forced liquidation. The notion of predatory trading partially overlaps with that of stock price manipulation, which is studied by Allen and Gale (1992) among others. One distinctive feature of predatory trading is that the predator derives profit from the price impact of the prey and not from his own price impact. Attari, Mello, and Ruckes (2002) and Pritsker (2003) are close in spirit to our paper. Pritsker (2003) also finds price overshooting in an example with heterogeneous risk-averse traders. Attari, Mello, and Ruckes (2002) focus, in a two-period model, on a distressed trader's incentive to buy in order to temporarily push up the price when facing a margin constraint, and a competitor's incentive to trade in the opposite direction and to lend to this trader. The systemic risk component of our paper is related to the literature on financial crisis. Bernardo and Welch (2002) provide a simple model of "financial market runs" in which traders join a run out of fear of having to liquidate before the price recovers.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 provides a preliminary result which simplifies the analysis. Section 4 derives the equilibrium and discusses the nature of predatory trading, with both single and multiple predators. Further, it shows how predation can drive an otherwise solvent trader into financial distress and create a crisis for the whole market. Section 5 studies the difference between orderly and distressed liquidation value. Section 6 considers the whole game including the build up of the traders' positions. Risk management and regulatory implications pertaining to disclosure of hedge funds as well as other extensions are discussed in Section 7. Detailed proofs are relegated to the appendix.

2 Model

We assume that the economy has two assets: a riskless bond and a risky asset. For simplicity, we normalize the risk-free rate to zero. The risky asset has an aggregate supply of $S > 0$ and a final payoff v at time T. Here, v is a random variable[4] with an expected value of $E(v) = \mu$. One can also view the risky asset as the payoff associated with an arbitrage strategy consisting of multiple assets. The price of the risky asset at time t is denoted by $p(t)$. The economy has two kinds of agents: large strategic traders (arbitrageurs) and long-term investors. We can think of the strategic traders as hedge funds and proprietary trading desks and the long-term investors as pension funds and individual investors.

The long-term investors are price-takers and have, at each point in time, an aggre-

[4]All random variables are defined on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$.

gate demand of

$$Y(p) = \frac{1}{\lambda}(\mu - p), \tag{1}$$

depending on the current price p. We assume that the demand curve is downward sloping. This assumption is consistent with empirical findings of Shleifer (1986), Chan and Lakonishok (1995), Wurgler and Zhuravskaya (2002) and others.[5] Theoretically, there are several mechanisms that can produce downward sloping demand curves including investor heterogeneity and risk aversion. For example, the demand curve is linearly downward sloping if each individual investor has constant absolute risk aversion and if the payoff v is normally distributed.

Strategic traders, $i \in \{1, 2, ..., I\}$, are risk-neutral and seek to maximize their expected profit. Each strategic trader is large, and hence, his trading impacts the equilibrium price. He therefore acts strategically and takes his price impact into account when submitting his orders. Each strategic trader i has a given initial endowment, $x^i(0)$, of the risky asset and he can continuously trade the asset. In particular, strategic trader i chooses his trading intensity, $a^i(t)$. Hence, his position, $x^i(t)$, in the risky asset at t is

$$x^i(t) = x^i(0) + \int_0^t a^i(\tau) d\tau. \tag{2}$$

We assume that each large strategic trader is restricted to hold

$$x^i(t) \in [-\bar{x}, \bar{x}], \tag{3}$$

and that $S > I\bar{x}$. These assumptions imply that the total capital of strategic investors is not sufficient to make the price equal to the expected value of the asset $E(v) = \mu$. We consider this case since we are interested in the strategic interactions that arise if large traders do not have sufficient capital. Unlimited holdings by strategic traders imply the trivial outcome that $p = \mu$.

Furthermore, large strategic traders are subject to a risk of financial crisis. Section 4.1 assumes that there is an exogenous set of agents in crisis, whereas Section 4.2 deals with the case of endogenous crisis. We denote the set of "distressed" traders in crisis by I^c and the set of unaffected traders, the "predators," by I^p. If a strategic trader is in financial crisis, he must liquidate his position in the risky asset. Formally,

$$i \in I^c \quad \Rightarrow \quad \begin{cases} a^i(t) \leq -\frac{A}{I} & \text{if } \; x(t) > 0 \text{ and } t > t_0 \\ a^i(t) = 0 & \text{if } \; x(t) = 0 \text{ and } t > t_0 \\ a^i(t) \geq \frac{A}{I} & \text{if } \; x(t) < 0 \text{ and } t > t_0. \end{cases} \tag{4}$$

This statement says that a trader in crisis must liquidate his position at least as fast as A/I. This is the fastest that an agent can liquidate without incurring temporary

[5] These papers dispute Scholes (1972), which concludes that the demand curve is almost flat.

price impact costs — to be discussed below — if all other traders are liquidating at the same time.[6]

The assumption of forced liquidation can be explained by (external or internal) agency problems. Bolton and Scharfstein (1990) shows that an optimal financial contract may leave an agent cash constrained even if the agent is subject to predation risk.[7] Also, limited capital for a certain strategy and forced liquidation can be the result of a company's risk management policy. We note that our results do not depend qualitatively on the nature of the troubled agents' liquidation strategy, nor do they depend on the assumption that such agents must liquidate their entire position. It suffices that a troubled large trader must reduce his position before time T.

The trading mechanism works in the following way. In each instant, all orders are executed simultaneously with the same priority. The market clearing price $p(t)$ satisfies the condition that supply equals demand. That is, $Y(p) + X(t) = S$, where X is the aggregate holding of risky assets by all strategic traders,

$$X(t) = \sum_{i=1}^{I} x^i(t). \tag{5}$$

Hence, the price is seen, using (1), to be

$$p(t) = \mu - \lambda (S - X(t)). \tag{6}$$

As long as the total net trade of the strategic traders is not too large, that is, as long as

$$\left| \sum_i a^i(t) \right| \leq A, \tag{7}$$

all orders in this instant are executed at the current price $p(t)$. If $|\sum_i a^i(t)| > A$, there is a temporary price impact of γ for the orders beyond A. More precisely, investor i incurs a cost of

$$G\left(a^i(t), a^{-i}(t)\right) := \gamma \max\left\{0, a^i - \overline{a}, \underline{a} - a^i\right\}$$

where $a^{-i}(t) := \left(a^1(t), \ldots, a^{i-1}(t), a^{i+1}(t), \ldots, a^I(t)\right)$ and where $\overline{a} = \overline{a}(a^{-i}(t))$ and $\underline{a} = \underline{a}(a^{-i}(t))$ are, respectively, the unique solutions to

$$\begin{aligned} \overline{a} + \sum_{j, j \neq i} \min\left(a^j, \overline{a}\right) &= A; \\ \underline{a} + \sum_{j, j \neq i} \max\left(a^j, \underline{a}\right) &= A. \end{aligned}$$

[6]We will see later that, in equilibrium, a troubled trader who must liquidate maximizes his profit by initially liquidating at this speed. Liquidating fast minimizes the costs of front-running by other traders.

[7]Even though Bolton and Scharfstein (1990) focuses on predation in product markets and not financial markets, their argument is general.

In words, $\overline{a}$ ($\underline{a}$) is the highest intensity with which trader i can buy (sell) without incurring the cost associated with a temporary price impact. Further, G is the product of the per-share cost, γ, multiplied by the number of shares exceeding $\overline{a}$ or $\underline{a}$. We assume for simplicity that the temporary price impact is large, $\gamma \geq \lambda I \bar{x}$.

This market structure can be interpreted as follows.[8] The limit order book consists of long-term investors' limit orders and it is not infinitely deep at the current price level. When the strategic traders' orders hit the limit order boom at a moderate speed, new limit orders flow in and the price walks up or down the demand curve. It takes time for new orders to enter the limit order book, and hence, if the strategic traders trade too fast, they have a temporary impact cost associated with hitting orders far away from the equilibrium price. This assumption is similar in spirit to the assumption of Longstaff (2001), who analyzes an optimal portfolio problem when trading strategies must be of bounded variation. Another interpretation is that strategic traders must search for long-term investors in order to trade. Duffie, Gârleanu, and Pedersen (2002) provides a search framework in a finance context.

Strategic trader i's objective is to maximize his expected wealth subject to the constraints described above. His wealth is the final value, $x^i(T)v$, of his stock holdings reduced by the cost, $a^i(t)p(t) + G$, of buying the shares, where G is the temporary impact cost. That is, a strategic trader's objective is:

$$\max_{a^i(\cdot)\in\mathcal{A}^i} E\left(x^i(T)v - \int_0^T [a^i(t)p(t) + G\left(a^i\left(t\right), a^{-i}\left(t\right)\right)]dt\right), \tag{8}$$

where $\mathcal{A}^i$ is the set of $\{\mathcal{F}_t^i\}$-adapted processes that satisfy (3) and (4). The filtration $\{\mathcal{F}_t^i\}$ represents trader i's information. We assume that each strategic trader learns the extent of his own temporary price impact and, at time t_0, he also knows which traders must liquidate. We consider both the case in which the size of any distressed trader's position is disclosed and the case in which it is not. Hence, $\{\mathcal{F}_t^i\}$ is generated by $I^p\, 1_{(t\geq t_0)}$ and by $G\left(a^i\left(t\right), a^{-i}\left(t\right)\right)$, and, in the case of disclosure, by $x^i(t_0)1_{(t\geq t_0)}$, $i \in I^c$. Note that the filtration does not include the price process. This is because of un-modelled noise in the supply of assets. We abstract from supply uncertainty for ease of exposition in the main text of the paper. In Appendix B we show that our equilibrium is a Perfect Bayesian Nash equilibrium in a more general economy with supply uncertainty in which traders observe prices.

Definition 1 *An equilibrium is a set of processes* $(a^1, \ldots, a^I)$ *such that, for each* i, a^i *solves (8), taking* $a^{-i} = (a^1, \ldots, a^{i-1}, a^{i+1}, \ldots, a^I)$ *as given.*

[8] We could assume other market structures and get similar results. For instance, we could assume that each trader has a (personal) constraint on his trading intensity.

3 Preliminary Analysis

In this section, we show how to solve a trader's problem. For this, we re-write trader i's problem (8) as a constant (which depends on $x(0)$) plus

$$\lambda S x^i(T) - \frac{1}{2}\lambda \left[x^i(T)\right]^2 - \lambda \int_0^T \left[a^i(t) X^{-i}(t) + G(a^i(t), a^{-i}(t))\right] dt,$$

where we use $E(v) = \mu$, the expression (6) for the price, the relation $\int_0^T a^i(t) x^i(t) dt = \frac{1}{2}\left[x(t)^2\right]_0^T$, and where we define

$$X^{-i}(t) := \sum_{j=1,\ldots,I,\, j\neq i} x^j(t). \tag{9}$$

Under our standing assumptions, it can be shown that any optimal trading strategy satisfies $x^i(T) = \bar{x}$ if trader i is not in distress. That is, the trader ends up with the maximum capital in the arbitrage position. Furthermore, it is not optimal to incur the temporary impact cost. That is, each trader optimally keeps his trading intensity within his bounds $\underline{a}$ and $\overline{a}$. Hence, we have the following result, which is useful in solving each trader's optimization problem and deriving the equilibrium.

Lemma 1 *A trader's problem can be written as*

$$\begin{aligned} &\min_{a^i(\cdot)\in\mathcal{A}^i} \int_0^T a^i(t) X^{-i}(t)\, dt \\ s.t.\quad & x^i(T) = x^i(0) + \int_0^T a^i(t)\, dt = \bar{x} \quad \text{if } i \in I^p \\ & a^i(t) \in \left[\underline{a}\left(a^{-i}(t)\right), \overline{a}\left(a^{-i}(t)\right)\right]. \end{aligned}$$

The lemma shows that the trader's problem is to minimize $\int a^i(t) X^{-i}(t) dt$, that is, to minimize his trading cost, not taking into account his own price impact. This is because the model is set up such that the trader cannot make or lose money based on the way his own trades affect prices. (For example, λ is assumed to be constant.) Rather, the trader makes money by exploiting the way in which the *other* traders affect prices (through X^{-i}). This distinguishes predatory trading from price manipulation.[9]

4 The Predatory Phase ($t \in [t_0, T]$)

We first consider the "predatory phase," that is, the period $[t_0, T]$ in which some strategic traders face financial distress. In Section 6, we analyze the full game including

[9] See, for instance, Allen and Gale (1992) for an example of price manipulation.

the "investment phase" $[0, t_0)$ in which traders decide the size of their initial (arbitrage) positions. We assume that each strategic large trader has a given position in the risky asset of $x(t_0) \in (0, \bar{x}]$ at time t_0. Furthermore, we assume for simplicity that there is "sufficient" time to trade, $t_0 + 2\bar{x}I/A < T$.

We proceed in two stages. Section 4.1 analyzes how strategic traders exploit the fact that others must liquidate their position and the implied price effects. Section 4.2 endogenizes agents' default and studies how traders may force others out of the market, possibly leading to a wide-spread crisis.

4.1 Exogenous Default

Here, we take as given the set, I^c, of distressed traders, and the common initial holding, $x(t_0)$, of all strategic traders. A distressed trader j sells, in equilibrium, his shares at constant speed $a^j = -A/I$ from t_0 until $t_0 + x(t_0)I/A$, and thereafter $a^j = 0$. This behavior is optimal, as will be clear later. This liquidation strategy is known, in equilibrium, by all the strategic traders.

The predators' strategies are more interesting. We first consider the simplest case in which there is a single predator, and subsequently we consider the case with multiple competing predators.

4.1.1 Single Predator ($I^p = 1$)

In the case with a single predator, the strategic interaction is simple: the predator, say i, is merely choosing his optimal trading strategy given the known liquidation strategy of the traders in crisis. Specifically, the distressed traders' total position, X^{-i}, is decreasing to 0, and it is constant thereafter. Hence, using Lemma 1, we get the following equilibrium.

Proposition 1 *With $I^p = 1$, the following describes an equilibrium:*[10] *Each distressed trader sells with constant speed A/I for $\tau = \frac{x(t_0)}{A/I}$ periods. The predator sells as fast as he can without causing a temporary price impact for τ time periods, and then buys back for $\bar{x}/A$ periods. That is,*

$$a^{i*}(t) = \begin{cases} -A/I & \text{for } t \in [t_0, t_0 + \tau) \\ A & \text{for } t \in [t_0 + \tau, t_0 + \tau + \frac{\bar{x}}{A}) \\ 0 & \text{for } t \geq t_0 + \tau + \frac{\bar{x}}{A}. \end{cases} \tag{10}$$

[10]The predator's profit does not depend on how fast he buys back his shares as long as he does not incur temporary impact costs and as long as he ends fully invested. Hence, there are other equilibria in which the predator buys back at a slower rate. These equilibria are, however, qualitatively the same as the one stated in the proposition, and there is no other equilibrium than these ones.

The price overshoots; the price dynamics is

$$p^*(t) = \begin{cases} p(t_0) - \lambda A[t - t_0] & \textit{for} \quad t \in [t_0, t_0 + \tau) \\ p(t_0) - \lambda I x(t_0) + \lambda A[t - (t_0 + \tau)] & \textit{for} \quad t \in [t_0 + \tau, t_0 + \tau + \frac{\bar{x}}{A}) \\ \mu + \lambda[\bar{x} - S] & \textit{for} \quad t \geq t_0 + \tau + \frac{\bar{x}}{A} \end{cases} \tag{11}$$

where $p(t_0) = \mu + \lambda(Ix(t_0) - S)$.

We see that although the surviving strategic trader wants to end up with all his capital invested in the arbitrage position ($x^i(T) = \bar{x}$), he is *selling* as long as the liquidating trader is selling. He is selling to profit from the price swings that occur in the wake of the liquidation. The predatory trader would like to "front-run" the distressed trader by selling before him and buying back shares after the distressed trader has pushed down the price further. Since both traders can sell at the same speed, the equilibrium is that they sell simultaneously and the predator buys back in the end. (The case in which predators can sell earlier than distressed traders is considered in Section 7.1.)

The selling by the predatory trader leads to price "overshooting." The price falls not only because the distressed trader is liquidating, but also because the predatory trader is selling as well. After the distressed trader is done selling, the predatory trader starts buying until he is at his capacity, and this pushes the price up towards its new equilibrium level.

The predatory trader is profiting from the distressed trader's liquidation for two reasons. First, the predator can sell his assets for an average price that is higher than the price at which he can buy them back after the distressed trader has left the market. Second, the predator can buy the additional units cheaply until he reaches his capacity. Hence, the predator has an incentive to try to cause the distressed trader's liquidation. We discuss in Section 4.2 how a predator may try to drive a vulnerable trader in bankruptcy by selling shares and causing a price drop.

The predatory behavior by the surviving agent makes liquidation excessively costly for the distressed agent. To see this, suppose a trader estimates the liquidity in "normal times," that is, when no trader is in distress. The liquidity — as defined by the price sensitivity to demand changes — is given by λ in Equation (6). When liquidity is needed by the distressed trader, however, the liquidity is lower due to the fact that the predator is selling as well. Specifically, the price moves by $I\lambda$ for each unit the distressed trader is selling. Endogenous liquidity and its pricing implications are discussed further in Section 5.

The distressed trader's excess liquidation cost equals the predator's profit from preying. Note that the predators do not exploit the group of long-term investors. The price overshooting implies that long-term investors are buying shares and selling them at the same price. Hence, it does not matter for the group of long-term investors whether the predator preys or not.[11]

Numerical Example We illustrate this predatory behavior with a numerical example. We let $I = 2$, $I^c = I^p = 1$, $\lambda = 1$, $\mu = 140$, $S = 40$, $t_0 = 5$, $T = 7$, $A = 20$, $x(t_0) = 8$, and $\bar{x} = 10$. Figure 1 illustrates the holdings of the distressed trader: This trader starts liquidating his position of 8 shares at time $t_0 = 5$ with a trading intensity of $A/2 = 10$ shares per time unit. He is done liquidating at time 5.8. At time 5, the predator knows that this liquidation will take place, and, further, he realizes that the price will drop in response. Hence, he wants to sell high and buy back low. The predator optimally sells all his 8 shares simultaneously with the distressed trader's liquidation, and, thereafter, he buys back 10 shares, reaching his capacity, as shown in Figure 2.

Figure 3 shows the price dynamics. The price is falling from time 5 to time 5.8 when both strategic traders are selling. Since 16 shares are sold and $\lambda = 1$, the price drops 16 points, falling to 100 below its long-run level of 110. Hence, there is a price overshooting of 10. As the predator re-builds his position from time 5.8 to time 6.3, the price recovers to 110.

It is intriguing that the predator is *selling* even when the price is below its long-run level 110. This behavior is optimal because, as long as the distressed trader is selling, the price will drop further and the predator can profit from selling additional shares and later repurchasing them. To further explain this point, we consider the predator's profit if he sells one share less. In this case, the predator sells 7 shares from time 5 to time 5.7, waits for the distressed trader to finish selling at time 5.8, and then buys 9 shares from time 5.8 to time 6.25. The price dynamics in this case is illustrated by the dotted line in Figure 4. We see that the 9 shares are bought back at the same prices as the *last* 9 shares were bought in the case where the predator continues to sell as long as the distressed trader does. Hence, to compare the profit in the two cases, we focus on the price at which the 10th (and last) share is sold and bought back. This share is sold at prices between 102 and 100, that is, with an average price of 101. It is bought back for prices between 100 and 101, that is, for an average price of 100.50. Hence, this "extra" trade is profitable, earning a profit of 101-100.50=0.50.

Our results are driven by the inability to trade a large position instantly and the price impact of a large position. The results do not, however, depend crucially on the specifics of our assumptions. The robustness of our results should be clear from the numerical example. Since the distressed trader's liquidation depresses the price, the predator has an incentive to sell high and buy low, and this action by the predator leads to price overshooting and costly liquidation.

[11] Long-term investors could profit from using a predatory strategy. We assume, however, that these investors do not have sufficient skills and information to do that.

Figure 1: Holdings of distressed trader

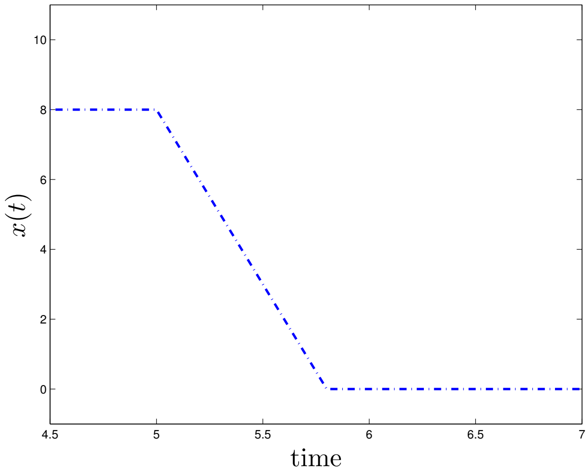


Figure 2: Holdings of predator

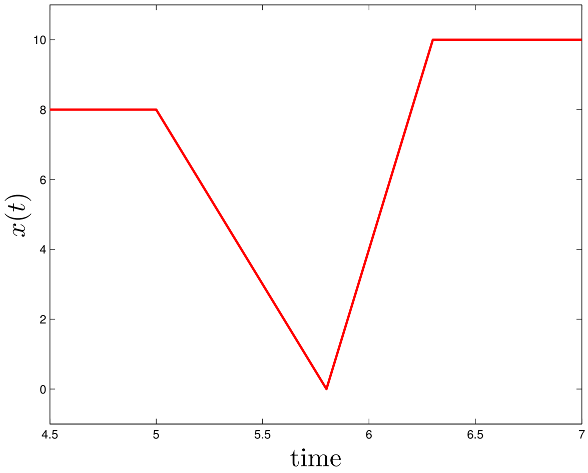


Figure 3: Price dynamics

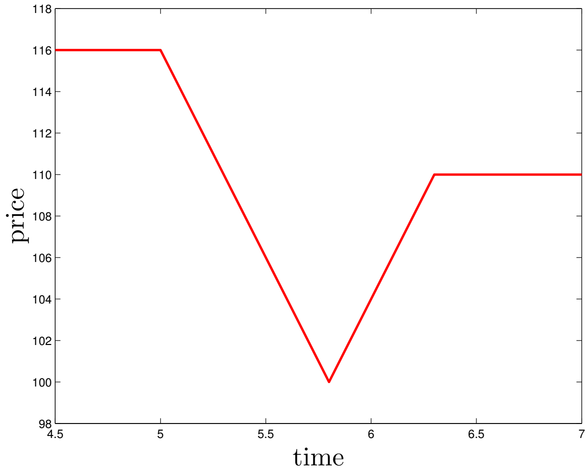
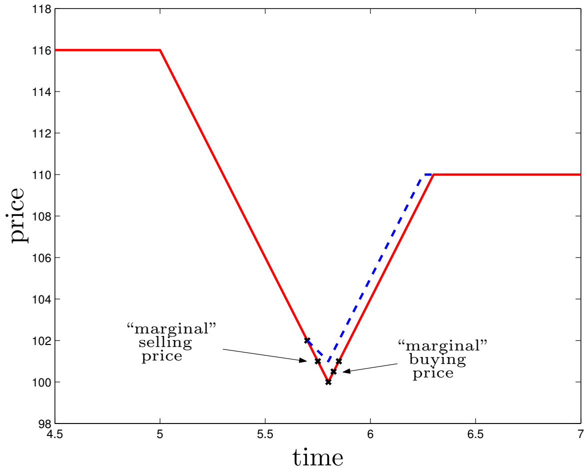


Figure 4: Price dynamics if predator sells 1 share less



4.1.2 Multiple Predators ($I^p \geq 2$)

We saw in the previous example how a single predatory trader has an incentive to "front run" the distressed trader by selling as long as the distressed trader is selling. With multiple surviving traders this incentive remains, but another effect is introduced: These predators want to end up with all their capital in the arbitrage position and they want to buy their shares sooner than the other strategic traders do.

The proposition below shows that, in equilibrium, predators sell for a while and then start buying back before the distressed traders have finished their liquidation.

Proposition 2 *With $I^p \geq 2$ and $x(t_0) \geq \frac{I^p-1}{I-1}\bar{x}$, in the unique symmetric equilibrium, each distressed trader sells with constant speed A/I for $\frac{x(t_0)}{A/I}$ periods. Each predator $i \in I^p$ sells at trading intensity A/I for $\tau := \frac{x(t_0)-\frac{I^p-1}{I-1}\bar{x}}{A/I}$ periods and buys back shares at a trading intensity of $\frac{AI^c}{I(I^p-1)}$ until $t_0 + \frac{x(t_0)}{A/I}$. That is,*

$$a^{i*}(t) = \begin{cases} -A/I & \text{for} \quad t \in [t_0, t_0+\tau) \\ \frac{AI^c}{I(I^p-1)} & \text{for} \quad t \in [t_0+\tau, t_0+\frac{x(t_0)}{A/I}) \\ 0 & \text{for} \quad t \geq t_0+\frac{x(t_0)}{A/I}. \end{cases} \tag{12}$$

The price overshoots; the price dynamics are

$$p^*(t) = \begin{cases} p(t_0) - \lambda A[t-t_0] & \text{for} \quad t \in [t_0, t_0+\tau) \\ p(t_0) - \lambda A\tau + \lambda\frac{AI^c}{I(I^p-1)}[t-(t_0+\tau)] & \text{for} \quad t \in [t_0+\tau, t_0+\frac{x(t_0)}{A/I}) \\ \mu + \lambda[\bar{x}I^p - S] & \text{for} \quad t \geq t_0+\frac{x(t_0)}{A/I}, \end{cases} \tag{13}$$

where $p(t_0) = \mu + \lambda I x(t_0) - \lambda S$.

The proposition shows that price overshooting also occurs in the case of multiple predators. This is because the predators' strategic behavior and limited capital make them sell excessively at first, and start buying relatively late. Note that, from each predator's perspective, $X^{-i}(t)$ (the stockholding of all other traders) is declining until $t_0+\tau$ and is constant thereafter. Since predator i also sells until $t_0+\tau$, aggregate stock holdings $X(t)$ and the price overshoot. The size of the overshooting is driven by trading activity of one predator. In other words, if hypothetically one predator refrained from preying and the others preyed using their equilibrium strategies, then there would be no overshooting.

The proposition states that the overshooting happens as long as traders' initial holding is large enough relative to the their position limit, that is, $x(t_0) \geq \frac{I^p-1}{I-1}\bar{x}$. Traders optimally choose such large positions if the risk of default is not too large as described carefully in Section 6. If, on the other hand, the traders' initial position, $x(t_0)$, is low, then there is no overshooting since the liquidated position is simply absorbed by the competing surviving predators. This follows from Proposition 2' in Appendix A.

The price overshooting is lower if there are more predators since more predators behave more competitively:

Proposition 3 *Keep the fraction, I^p/I, of predators, the total arbitrage capacity, $I\bar{x}$, and the total initial stock holding, $Ix(t_0)$ constant, and assume that $Ix(t_0) \geq I^p\bar{x}$. The price overshooting*
(i) is strictly positive for all finite I^p;
(ii) is decreasing in the number of predators I^p; and
(iii) approaches 0 as I^p approaches infinity.

Numerical Example We consider the cases with a total number of traders $I = 3$, 9, and 27. For each case, we assume that a third of the traders are in crisis, that is, $I^c/I = 1/3$. As in the previous example, we let $\lambda = 1$, $\mu = 140$, $S = 40$, $t_0 = 5$, $T = 7$, the total trading speed $A = 20$, the total initial holding $x(t_0) \cdot I = 16$, and the total trader holding capacity $\bar{x} \cdot I = 20$. Figure 5 illustrates the holdings of predatory traders and Figure 6 shows the price dynamics.

We see that there is a substantial price overshooting when the number of predators is small, and that the overshooting is decreasing as the number of predators increases. More predators increase the competitive pressure to buy shares early. Hence, the liquidation cost for a distressed trader is decreasing in the number of predators (even holding the total trading capacity fixed).

We note a striking difference between the case of one predator and the case with several predators. With a single predator, the predator keeps selling until the distressed traders have finished their liquidation, whereas with several predators the competitive pressure makes the predators buy back earlier and finish buying back at the same time as the distressed traders have finished their selling.

Collusion. The predators can profit from collusion. In particular, they could increase their revenue from predation by selling until the troubled traders were finished liquidating and only then start rebuilding their positions. Hence, through collusion, the predators could jointly act like a single predator (with the slight modification that multiple predators have more capital). Collusive and non-collusive outcomes are qualitatively different. A collusive outcome is characterized by predators buying shares only after the troubled traders have left the market and by a large price overshooting. In contrast, a non-collusive outcome is characterized by predators buying all the shares they need by the time the troubled traders have finished liquidating and by a relatively smaller price over-shooting.

Collusion could potentially occur through an explicit arranged agreement or implicitly without arrangement, called "tacit" collusion. Tacit collusion means that the

Figure 5: Holdings, $x^i(t)$, of each predator

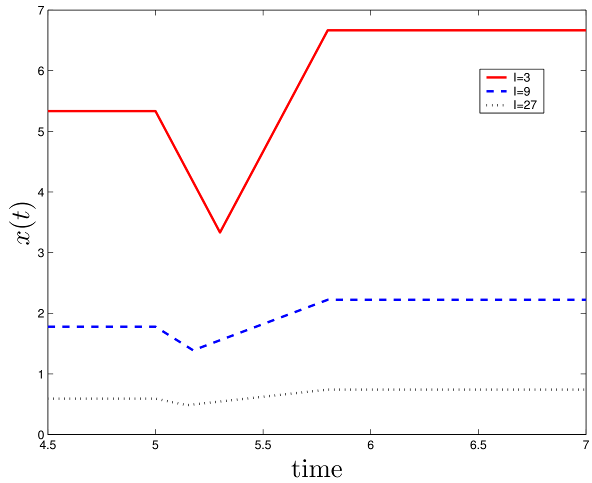
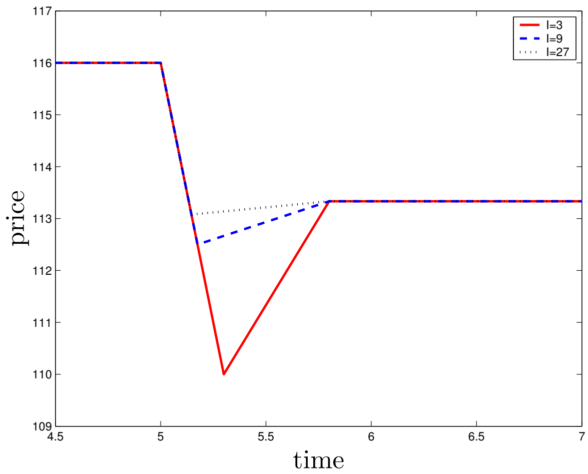


Figure 6: Price dynamics



collusive outcome is the equilibrium in a non-cooperative game. In our model, tacit collusion cannot occur. However, if strategic traders could observe (or infer) each others' trading activity, then tacit collusion might arise because predators could "punish" a predator that deviates from the collusive strategy.[12]

4.2 Endogenous Default and Systemic Risk

So far we have assumed that a certain set of large strategic traders exogenously fall into financial distress, without specifying the underlying cause. In this section, we endogenize distress and study how predatory activity can lead to contagious default events. If a trader's wealth drops to a threshold level $\underline{W}$, he must liquidate. This is because of margin constraints, risk management, or other considerations in connection with low wealth. Trader i's wealth at t consists of his position $x^i(t)$ of the asset that our analysis focuses on, as well as wealth held in other assets $O^i(t)$. That is, his "paper wealth" is $W^i(t) = x^i(t)\,p(t) + O^i(t)$. The value of the other holdings $O^i(t)$ is subject to an exogenous shock at time t_0, which can be observed by all traders. Thereafter, $O^i(t)$ stays constant.

Obviously, if the wealth shock ΔO^i at t_0 is so large that $W^i(t_0) \leq \underline{W}$, the trader is immediately in distress and must liquidate. Smaller negative shocks with $W^i(t_0) > \underline{W}$ can, however, also lead to an "endogenous default," since the potential selling behavior of predators and other distressed traders may erode the wealth of trader i even further.

[12]If traders could observe each others' trades, then we would have to change our definition of strategies and equilibrium accordingly. A rigorous analysis of such a model is beyond the scope of this paper.

That is, predation can drive other traders into bankruptcy. A trader who knows that he must liquidate in the future finds it optimal to start selling already at time t_0. The set of traders who anticipate having to liquidate is denoted by I^c as in previous sections. Interestingly, whether an agent anticipates having to liquidate depends on the number of other agents who are expected to be in crisis.

We let $\underline{W}(I^c)$ be the maximum wealth at t_0 such that trader i cannot avoid financial crisis if I^c traders are *expected* to be in crisis. More precisely, for $I^c > 0$, it is the maximum wealth such that, for any feasible strategy it holds that $\min_{t\in[t_0,T]} W^i(t, a^i, a^{-i}) \leq \underline{W}$ where a^{-i} has $(I^c - 1)$ strategies of liquidating and I^p strategies of preying. Further, for $I^c = 0$, $\underline{W}(0) = \underline{W}$.

With this definition of $\underline{W}(I^c)$, it follows directly that — in an equilibrium[13] in which I^c traders immediately liquidate and $I^p = I - I^c$ traders prey as in Propositions 1, 2 and 2' — every distressed trader $i \in I^c$ has wealth $W^i(t_0) \leq \underline{W}(I^c)$, and every predator $i \in I^p$ has $W^i(t_0) > \underline{W}(I^c)$.

Interestingly, the higher is the expected number, I^c, of defaulting traders, the higher is the "survival hurdle" $\underline{W}(I^c)$.

Proposition 4 *The more traders are expected to be in crisis, the harder it is to survive. That is, $\underline{W}(I^c)$ is increasing in I^c.*

This insight follows from the fact that a higher number of defaulting traders make predation more fierce since there are fewer competing predators and more prey to exploit. This fierce predation lowers the price at all times, making survival more difficult.

Proposition 4 is useful in understanding systemic risk. Financial regulators are concerned that the financial difficulty of one or two large traders can drag down many more investors, thereby destabilizing the whole economy. Our framework helps explain why this spillover effect occurs. To see this, consider the economy depicted in Figure 7. Trader A's wealth is in the range of $(\underline{W}(1), \underline{W}(2)]$, trader B's wealth is in $(\underline{W}(2), \underline{W}(3)]$, and trader C's is in $(\underline{W}(3), \underline{W}(4)]$. The three remaining traders (D, E, and F) have enough reserves to fight off any crisis, that is, their wealth is above $\underline{W}(I)$.

With these wealth levels, the unique equilibrium is such that no strategic trader is in distress and all of them immediately start increasing their position from $x(t_0)$ to $\bar{x}$. To see this, note first that it cannot be an equilibrium that one agent defaults. If one agent is expected to default, no one defaults because no one has wealth below $\underline{W}(1)$. Similarly, it is not an equilibrium that two traders default, because only trader A has wealth below $\underline{W}(2)$, and so on.

[13]There may be other kinds of equilibria in which some surviving traders do not prey because of fear of driving themselves in distress. For ease of exposition, we do not consider these equilibria. Equilibria of the form that we consider exist under regularity conditions on the initial holdings and wealths.

Figure 7: No trader is in distress.

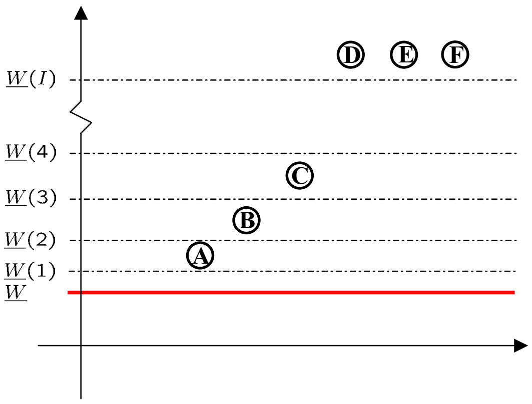
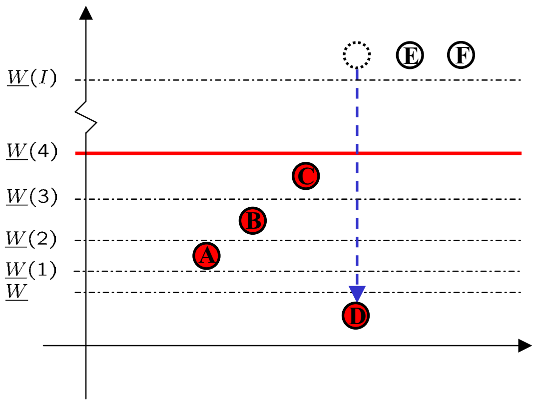


Figure 8: One trader drags three down.



On the other hand, if trader D faces a wealth shock at t_0 such that $W^D(t_0) < \underline{W}$, he can drag down traders A, B, and C, as shown in Figure 8. If it is expected that four traders will be in distress, then traders A, B, C, and D will liquidate their position since their wealth is below $\underline{W}(4)$. Intuitively, the fact that trader D is forced to liquidate his position encourages predation and the price is depressed. This, in turn, brings three other traders into financial difficulty. This situation captures the notion of systemic risk. The financial difficulty of one trader endangers the financial stability of three other traders.

There are also other equilibria in which 1, 2, or 3 traders face distress. For instance, it is an equilibrium that only trader D liquidates, since if other traders do not panic and everybody expects that only trader D will go under, traders A, B, C, E, and F prey and buy back after a short while. The predation is less fierce in this equilibrium in the sense that predators start re-purchasing shares earlier (i.e., turning point, $t_0 + \tau$, occurs earlier).

It is important to notice that the multiplicity in our example does not arise when trader E also faces a wealth shock at t_0 such that $W^E(t_0) < \underline{W}(1)$. In this case, at least two traders must liquidate, which drives A in default since A has wealth less than $\underline{W}(2)$. Hence, at least three traders must liquidate, which makes predation yet fiercer and drives B in default. Similarly, this results in C's default, and we see that the "ripple-effect" equilibrium is unique in this case.

In our perfect information setting, all traders know, the instant after t_0, how the equilibrium will play out. That is, they know the entire future price path as well as the number of predators I^p and victims I^c. In a more complex setting in which traders' wealth shocks are not perfectly observable and the price process is noisy, this need not be the case. A trader might start selling shares not knowing when the price decline stops. He might expect to act as a predator but may actually end up as prey.

In the case of multiple equilibria, coordination by predators might lead to more predation, while coordination by potential victims might prevent a financial crisis. For example, coordination among predators is required if two or more predators are needed to push the price sufficiently down to drive a third trader into financial distress. On the other hand, coordination among vulnerable traders might prevent predation. For example, one can imagine a situation in which the wealth level of two of three strategic traders drops into the range $(\underline{W}, \underline{W}(2)]$ such that if both these traders stay put, their wealth level never falls below $\underline{W}$. Thus, the predator will not attack and all traders survive. On the other hand, if the vulnerable traders start selling along with the predator, they drive each other into distress since the price decline erodes their wealth to a level below $\underline{W}$. In other words, their panic selling behavior helps the predator to exploit the situation.

The dangers of systemic risk in financial markets provide an argument for intervention by regulatory bodies, such as central banks. A bailout of one or two traders or even only a coordination effort can stabilize prices and ensure the survival of numerous other vulnerable traders. However, it also spoils the profit opportunity for the remaining predators who would otherwise benefit from the financial crisis. From an ex-ante perspective, the anticipation of crisis preventive action by the central bank reduces the systemic risk of the financial sector, and hence, traders are more willing to exploit arbitrage opportunities. This reduces the initial mispricing, but it could also worsen agency problems not considered here.

Finally, while in our equilibrium all vulnerable traders start liquidating their position from t_0 onwards, one sometimes observes that these traders miss the opportunity to reduce their position early. This exacerbates the predation problem, since a delayed reaction on the part of the distressed traders allows the predators to front-run as discussed in Section 7.1. The phenomenon of delayed reaction by vulnerable traders may be explained in an enriched version of our framework. First, if prices are fluctuating, the trader might "gamble for resurrection" by not selling early, in the hope that a positive price shock will liberate him from financial distress. Second, if selling activity cannot be kept secret, a desire to appear solvent might prevent a troubled trader from selling early.

5 Valuation with Endogenous Liquidity

Predatory trading has implications for valuation of large positions. We consider three levels of valuation with increasing emphasis on the position's liquidity.

Definition 2 *(i) The "paper value" of a position x at time t is $V^{paper}(t,x) = xp(t)$;*
(ii) the "orderly liquidation value" is $V^{orderly}(t,x) = x\left[p(t) - \frac{1}{2}\lambda x\right]$; and
(iii) the "distressed liquidation value", $V^{distressed}(t,x,I^p)$, is the revenue raised in equilibrium when I^p predators are preying.

The paper value is the simple mark-to-market value of the position. The orderly liquidation value is the revenue raised in a secret liquidation, taking into account the fact that the demand curve is downward sloping. The downward sloping demand curve implies that liquidation makes the price drop by λx, resulting in an average liquidation price of $p(t) - \frac{1}{2}\lambda x$.

The distressed liquidation value takes into account not only the downward sloping demand curve, but also the strategic interaction between traders and, specifically, the costs of predation. We note that $V^{distressed}$ depends on the characteristics of the market such as the number of predators, the number of troubled traders, and their initial holdings. For instance, the distressed valuation of a position declines if other traders also face financial difficulty.

Clearly, the orderly liquidation value is lower than the paper value. The distressed liquidation value is even lower if the predators have initially large positions.[14]

Proposition 5 *If $x(t_0) \geq \frac{\sqrt{I^p(I^p-1)}}{I-1}\bar{x}$, then the paper value of the position is greater than the orderly liquidation value, which in turn is greater than the distressed liquidation value. That is, $V^{paper}(x(t_0)) > V^{orderly}(x(t_0)) > V^{distressed}(x(t_0))$.*

The low distressed liquidation value is a consequence of predation. In particular, predation causes the price to initially drop much faster than what is warranted by the distressed trader's own sales. Hence, the market is endogenously more illiquid when a distressed trader needs liquidity the most.

Consider a strategic trader estimating the liquidity of the market in "normal" times. This liquidity estimate leads to an estimate of the liquidation value of $V^{orderly}$. The actual liquidation value in the case of distress, however, can be much lower.

It is interesting to consider what happens as the number of predators grows, keeping constant their total predation capacity. More predators implies that their behavior is more similar to that of a price-taking agent. This more competitive behavior makes predation less fierce and increases the distressed liquidation value.

Importantly, even in the limit with infinitely many predators, the distressed liquidation value is strictly lower than the orderly liquidation value. This is because the price drops faster when there are more predators, and this leads to a reduction in the liquidation value. Recall, in contrast, that Proposition 3 shows that the price overshooting disappears as the number of predators grow. It is, however, not only the price overshooting that reduces the distressed liquidation value.

Proposition 6 *Keep constant the fraction, I^p/I, of predators, the total arbitrage capital, $I\bar{x}$, and the total initial holding, $Ix(t_0)$, and suppose that $Ix(t_0) \geq \sqrt{I^p I}\bar{x}$. Then,*

[14] If the predators' initial position is low, then the distressed liquidation value can be greater than the orderly liquidation value. This is because the public announcement of a distressed liquidation can, in this case, cause the predators to start buying shares early. This buying behavior can make the public liquidation more profitable than a secret (orderly) liquidation.

the total distressed liquidation value, $I^c V^{distressed}$ is increasing in the number of predators, I^p. In the limit, as I^p approaches infinity, the total distressed liquidation revenue remains strictly smaller than the total orderly liquidation value,
$\lim_{I^p \to \infty} I^c V^{distressed}(x(t_0)) < V^{orderly}(I^c x(t_0))$.

One could further apply our framework to study the *ex-ante* value of a large position (and the expertise in trading in this market), taking into account the risk of predation against oneself and the possible rewards from predation against others. This would be relevant, for instance, when considering a take-over of a hedge fund.

6 The Investment Phase ($t \in [0, t_0)$)

So far, we have taken the number, $x(t_0)$, of shares that traders hold before predation starts as given. In this section, we analyze the process of acquisition of the initial position prior to t_0. The strategic traders' initial position at time 0 — when they learn of the arbitrage opportunity — is assumed to be zero. To separate the investment phase from the predatory phase, we assume that t_0 occurs sufficiently late, that is, $t_0 > \frac{\bar{x}}{A/I}$. This ensures that traders can acquire any position $x \in [-\bar{x}, \bar{x}]$ prior to t_0. We focus, in this section, on the case in which at most one strategic trader faces financial crisis. Specifically, we assume that with probability π, I^c is a singleton with all traders having equal probability of being in crisis, and with probability $1 - \pi$, no trader is in crisis, that is, $I^c = \emptyset$.[15] Proposition 7 describes the initial trading by large strategic investors.

Proposition 7 *First, all traders buy at the rate A/I until they have accumulated a position of $x(t_0)$. If $I > 2$ and a distressed trader's position is not disclosed, then*

$$x(t_0) = \left(1 - \frac{\pi}{I}\right) \bar{x}.$$

If $I = 2$ or if a distressed trader's position is disclosed, then

$$x(t_0) = \left(1 - \frac{\pi}{I-1}\right) \bar{x}.$$

After time t_0, traders use the (predatory) strategies described in Propositions 1, 2, and 2'.

All traders try to acquire their desired position $x(t_0)$ as quickly as possible. This is because delay is costly, since other traders' acquisitions increase the price. Importantly,

[15] Here, we make (implicitly) the simplifying assumption that the default risk does not depend on the size of the position.

it is this desire of the traders to quickly acquire a large position that later leaves them vulnerable to predation.

The optimal pre-t_0 position, $x(t_0)$, balances the costs and benefits associated with the three possible outcomes: (i) no trader faces crisis, (ii) another trader faces crisis, and (iii) the trader himself is in crisis. First, with probability $1-\pi$, no strategic trader faces financial distress before time T. In this case, all traders try to acquire additional shares up to their maximum position $\bar{x}$ after time t_0. A trader who acquired a larger position prior to t_0 has the advantage that he purchased it at a lower price.

Second, with probability $\pi\frac{I-1}{I}$, one of the other traders faces financial difficulty and must liquidate his position. In this case, it is advantageous for a surviving trader to have a smaller position $x(t_0)$ since it is cheaper to acquire the shares after t_0.

Finally, with probability $\frac{\pi}{I}$, the trader himself must liquidate his position. In this case, the effect of having initially bought — out of equilibrium — a large position depends on whether this position is disclosed. If the distressed trader's position is made public, then it is always worse, in case of distress, to have bought a larger position. This is because the predation makes the liquidation very costly.

If a distressed trader's position is not disclosed and if $I > 2$, then the effect of initially buying some extra shares may be reversed because the initial purchase is secret. This explains why the initial position is larger in the case of no disclosure ($1-\frac{\pi}{I} > 1-\frac{\pi}{I-1}$). More general implications of disclosure are discussed in Subsection 7.4.

The default and predation risks limit the traders' willingness to build a large arbitrage position, that is, $x(t_0) < \bar{x}$. It is interesting to compare the total initial arbitrage position, $Ix(t_0)$, with the expected surviving arbitrage capital. The expected number of survivors is $I-\pi$, so the expected surviving arbitrage capital is $\bar{x}(I-\pi)$.

If $I = 2$ or if there is disclosure, the initial total arbitrage position, $Ix(t_0) = \bar{x}(I-\pi\frac{I}{I-1})$, is lower than the expected surviving arbitrage capital. This is because of the large cost associated with selling (an extra unit) to a monopolist predator or to predators who know that you bought the extra shares. Hence, the risk of costly predation limits the arbitrage positions, and this initially makes the price depart further away from the expected value of the asset.

With $I > 2$ and if there is no disclosure, the initial total arbitrage position is equal to the expected surviving arbitrage capital, that is, $Ix(t_0) = \bar{x}(I-\pi)$. This leads to a "martingale-like" property of the price. This is because, in case one defaults, the marginal share is sold at a "fair" price when all the predators have bought back their position. If the risk of default was increasing in the size of the position, however, then arbitrage positions would initially be smaller.

7 Further Implications of Predatory Trading

7.1 Front-running

So far, we have considered equilibria in which the distressed traders sell at the same time as the predators. Anecdotal evidence suggests that, in some cases, the predators are selling *before* the distressed trader. That is, the predators are truly front-running. There are various potential reasons for the delayed selling by the distressed traders. They might hope that they will face a positive wealth shock that will allow them to overcome the financial difficulty and avoid liquidation and default costs. Alternatively, the distressed trader may not be aware that the predator — for instance, the trader's own investment bank — is preying on him. Finally, the predators could simply have an ability to trade faster. In any case, front-running makes predation even more profitable.

The equilibrium with a single predator is simple. First, the predator sells as much as possible. Then, he waits for the distressed trader to depress the price by liquidating his position, and finally the predator repurchases his position. Clearly, the price overshoots and the predation makes liquidation costly.

The equilibrium with many predators can also easily be analyzed within our framework. Suppose that, at time t_0, it is clear that I^c traders are in financial distress, and that these traders start selling at time t_1, where $t_1 > t_0 + \frac{I^c I^p}{A(I^p-1)}\overline{x}$.

The predatory trading plays out as follows. First, the predators front-run by selling. This leads to an excessively large price drop. When the distressed traders start selling, the predators are buying back, and the price recovers to its new equilibrium level.

Proposition 8 *With $I^p \geq 2$ and $x(t_0) \geq \frac{I^p-1}{I-1}\bar{x}$, in the unique symmetric equilibrium, each distressed trader sells with constant speed A/I for $\frac{x(t_0)}{A/I}$ periods starting at time t_1. Each predator $i \in I^p$ starts selling from t_0 onwards at trading intensity A/I^p for $\tau := \frac{(I-1)x(t_0)-\bar{x}(I^p-1)}{A(I^p-1)/I^p}$ periods and buys back shares at a trading intensity of $\frac{A}{I}\frac{I^c}{I^p-1}$ from t_1 onwards. That is,*

$$a^{i*}(t) = \begin{cases} -A/I^p & \text{for } \; t \in [t_0, t_0+\tau) \\ 0 & \text{for } \; t \in [t_0+\tau, t_1) \\ \frac{AI^c}{I(I^p-1)} & \text{for } \; t \in [t_1, t_1 + \frac{x(t_0)}{A/I}) \\ 0 & \text{for } \; t \geq t_1 + \frac{x(t_0)}{A/I}. \end{cases} \tag{14}$$

The price overshoots; the price dynamics is

$$p^*(t) = \begin{cases} p(t_0) - \lambda A[t-t_0] & \text{for } \; t \in [t_0, t_0+\tau) \\ p(t_0) - \lambda A\tau & \text{for } \; t \in [t_0+\tau, t_1) \\ p(t_0) - \lambda A\tau + \lambda \frac{A}{I}\frac{I^c}{I^p-1}[t-t_1] & \text{for } \; t \in [t_1, t_1 + \frac{x(t_0)}{A/I}) \\ \mu + \lambda[\bar{x}I^p - S] & \text{for } \; t \geq t_1 + \frac{x(t_0)}{A/I}. \end{cases} \tag{15}$$

where $p(t_0) = \mu + \lambda I x(t_0) - \lambda S$. *The ability to front-run by predators imply larger liquidation costs for distressed traders and greater price overshooting.*

Changes in the composition of main stock indices force index funds to re-balance their portfolios to minimize their tracking errors. While prior to 1989 changes in the composition of S&P occurred without prior notice, from 1989 onwards they were announced one week in advance. The price dynamics during these intermediate weeks suggest index that tracking funds re-balance their portfolio around the inclusion/deletion date, while strategic traders front-run by trading immediately after the announcement. Lynch and Mendenhall (1997) documents a sharp price rise (drop) on the announcement day, a continued rise (decline) until the actual inclusion (deletion), and a partial price reversal on the days following the inclusion (deletion). Hence, consistent with our model's predictions, there appears to be front-running and price overshooting. The highest volume occurs on the day prior to the inclusion (deletion), indicating that most of the index funds trade on this day. If so, the empirical observations are not fully in line with the model, since it predicts that the reversal starts already on this day (not the following day as in the data), unless there is a monopolist strategic trader or traders collude.

7.2 Batch Auction Markets, Trading Halts, Circuit Breakers

The degree of price overshooting and predatory behavior depends on the market structure. In this subsection, we consider a setting in which orders are batched together in a call auction market immediately after some traders are forced to sell their holdings at t_0. We assume that distressed traders must sell their entire holding at the batch auction. Their orders are batched with the remaining predators' market orders and the long-term traders' continuum of limit orders. After all orders are collected, they are executed at a single price. This prevents the predators from walking down the demand curve. After the batch auction sequential trading resumes and the market structure is as before.

Note that trading halts and circuit breakers work exactly this way. Trading is suspended for some time and orders are collected. Trading recommences with the standard opening procedure which is organized as a batch auction on most stock exchanges. Continuous trading resumes after the new opening price is found.

Proposition 9 describes the equilibrium behavior of the predators and the price dynamics for this setting.

Proposition 9 *With* $x(t_0) \geq \frac{I^p-1}{I-1}\bar{x}$, *each predator submits a buy order of size* $\frac{I^p-1}{I^p}\left[\overline{x} - x(t_0)\right]$ *at the batch auction at* t_0. *Thereafter, each predator buys at a trading intensity of* A/I^p *for* $\left[\overline{x} - x(t_0)\right]/A$ *periods.*

The price dynamics is

$$p^*(t) = \begin{cases} \mu - \lambda S + \lambda (I^p - 1)\overline{x} + \lambda x(t_0) & \text{at the} \quad \text{batch auction at } t_0 \\ \mu - \lambda S + \lambda (I^p - 1)\overline{x} + \lambda x(t_0) + \lambda A[t - t_0] & \text{for} \quad t \in [t_0, t_0 + \frac{\overline{x} - x(t_0)}{A}) \\ \mu - \lambda S + \lambda I^p \bar{x} & \text{for} \quad t \geq t_0 + \frac{\overline{x} - x(t_0)}{A}. \end{cases} \tag{16}$$

The price overshooting is smaller compared to the setting without batch auction.

In contrast to the sequential market structure, predators do not sell shares. However, they are still reluctant to provide liquidity as long as competitive forces are weak. To see this, note that a single predator will not participate in the batch auctions at all, while in the case of multiple predators each individual predator's order size is limited to $\frac{I^p-1}{I^p}[\overline{x} - x(t_0)]$. This explains why some price overshooting remains. After the batch auction, the surviving predators build up their final position $x(T) = \overline{x}$ as fast as possible in continuous trading. Hence, the price gradually increases until it reaches the same long-run level $p(T) = \mu - \lambda S + \lambda I^p \bar{x}$. The price overshooting is substantially lower compared to the sequential trading mechanism and the new long-run equilibrium price is reached more quickly.

7.3 Risk Management

Predatory trading amplifies default risk. Hence, a detailed examination of predation risk should be one pillar in any risk management analysis. This is especially the case for large traders, such as hedge funds, who often hold illiquid assets.

One lesson which emerges from this paper is that risk management strategies that follow publicly known mechanical rules invite predatory trading by other traders. In general, granting fund managers more flexibility and making the trading strategy less stringent reduces predation risk, but it also limits the control over the manager. Examples of such mechanical rules include dynamic hedging strategies like portfolio insurance trading. Viewed from this angle, our mechanism provides a fresh perspective on the stock market crash of 1987. An initial price decline forced many portfolio insurance traders to sell their shares.[16] Other strategic traders who were aware of this fact preyed on them and hence exacerbated the fall. Consistent with this result, the Brady Report (Brady et al. (1988), p. 15) states:

> ... This precipitous decline began with several "triggers," which ignited mechanical, price-insensitive selling by a number of institutions following

[16]Gennotte and Leland (1990) also studies the crash in the context of portfolio insurance. They assume that a large fraction of investors are less informed, not aware that the initial selling was due to non-informative liquidation needs, similar to our long-term investors. While they consider a static model, our paper considers the dynamics.

> portfolio insurance strategies and a small number of mutual fund groups. The selling by these investors, and the prospect of further selling by them, encouraged a number of aggressive trading-oriented institutions to sell in anticipation of further declines. These aggressive trading-oriented institutions included, in addition to hedge funds, a small number of pension and endowment funds, money management firms and investment banking houses. This selling in turn stimulated further reactive selling by portfolio insurers and mutual funds. ...

While a fund's mechanical trading rules increase the risk that others prey on it, "predation risk" is magnified if many large traders follow similar risk management strategies. In the event of a wealth decline, fewer predators are left in the market, which exacerbates predation and the systemic risk component. Consequently, each fund's optimal risk management strategy depends on other traders' risk management in addition to the liquidity of the acquired assets.

Risk analysis should also take into account that predation can discourage a large trader from seeking outside financing to bridge temporary financial losses. This is because the trader may have to reveal its positions and trading strategy to the possible creditors, such as the trader's brokers, exposing the trader to predatory trading.

Many loans are secured by collateral assets, which the broker can sell at short notice if margin requirements are not met. Such fire-sales depress the selling price and increases the fund's (possible temporary) financial distress. An illustrative example is the case of Granite Partners (Askin Capital Management) whose main brokers — Merrill Lynch, DLJ and others — gave the fund less than 24 hours to meet a margin call. Merrill Lynch and DLJ then allegedly sold off collateral assets at below market price at an insider-only auction where bids were solicited from a restricted number of other brokers excluding retail institutional investors.

Our analysis further implies that large traders can benefit from dealing with multiple brokers and banks to reduce the amount of sensitive information known by each counterparty and to ensure that possible predatory trading is not too fierce or monopolistic.

7.4 Disclosure

Enforcing strict disclosure rules concerning a fund's security positions or risk management strategy increases "predation risk" in illiquid markets, and hence, can undermine the objective of the fund's investors. On the other hand, if there is much "available" arbitrage capital (i.e., other strategic traders have low current positions, $x(t_0)$, relative to their limits, $\bar{x}$), then it can be beneficial to publicly announce a liquidation since this will enhance competition to acquire the position (Proposition 2'). Also, disclosure improves monitoring of the fund manager. The optimal balance for this trade-off depends on the liquidity of the position, the fund's size, and the fund's agency problems.

IAFE Investor Risk Committee (IRC) (2001) argues along similar lines by favoring less detailed disclosure for large funds. The predatory risk of disclosure is reduced if it concerns the portfolio's characteristics rather than the specific positions, or if the disclosure is delayed in time. Second, our analysis suggests that any disclosed information should be dispersed as broadly as possible, in order to minimize the implications of predatory trading.

We have shown in Section 6 that disclosing the position of a distressed trader also alters the his ex-ante incentives to take on (arbitrage) positions. The fear of being preyed upon makes strategic traders less aggressive, and hence, larger mispricings can remain.

7.5 Bear Raids and the Uptick Rule

A bear raid is a special form of predatory trading, which was not uncommon prior to 1933 according to Eiteman, Dice, and Eiteman (1966).[17] A ring of traders identifies and sells short a stock that other investors hold long on their margin accounts. This depresses the stock's price and triggers margin calls for the long investors, who are then forced to sell their shares, further deflating the price. Based on his (allegedly first-hand) knowledge of these practices, Joe Kennedy, the first head of the SEC, introduced the so-called up-tick rule to prevent bear raids. This rule bans short-sales during a falling market. In the context of our model, this means that strategic traders with small initial positions, $x(t_0)$, cannot undertake predatory trading. This reduces the price overshooting and increases the distressed traders' liquidation revenue.

7.6 Contagion

Predatory trading suggests a novel mechanism for financial contagion. Suppose that the strategic traders have large positions in several markets. Further, suppose that a large strategic trader incurs a loss in one market, bringing this trader in financial trouble. Then, this large trader must downsize his operations. Hence, he reduces all his positions. As explained by our model, other traders have an incentive to undertake predatory trading, thereby enhancing the price impact in all affected markets.

This type of contagion is not driven by a correlation in economic fundamentals or by information spillovers but, rather, by the composition of the holdings of large traders who must significantly reduce their positions. This insight has the following empirical implication: a shock to one security, which is held by large vulnerable traders, may be contagious to other securities that are also held by the vulnerable traders.

[17] The origin of the term "bear" goes back to the 18th century, where it described a trader who sold the bear's skin before he had caught it.

8 Conclusion

This paper provides a new framework for studying the phenomenon of predatory trading. Predatory trading is important in connection with large security trades in illiquid markets. We show that predatory trading leads to price overshooting and amplifies a large trader's liquidation cost and default risk. Hence, "predation risk" should be a primary component of risk management. Predatory trading introduces systemic risk since a financial shock to one trader may spill over and trigger a crisis for the whole financial sector. Consequently, our analysis provides an argument in favor of coordinated actions by regulators or bailout. Our analysis has further implications for the regulation of securities trading and of large strategic traders, and it explains the advantages of circuit breakers and of the up-tick rule.

A Proofs

A.1 Proof of Proposition 1

The distressed trader's strategy is optimal since any faster liquidation leads to temporary price impact costs.

The surviving trader, i, wants to minimize $\int_{t_0}^{T} a^i(t)\, X^{-i}(t)\, dt$ subject to the constraints that $x^i(T) = \bar{x}$ and $|a^i(t)| \leq A/I$. Here, $X^{-i}(t)$ is the position of the trader in financial trouble so

$$X^{-i}(t) = \begin{cases} x(t_0) - t\, A/I & \text{for} \quad t \in [t_0, t_0 + x(t_0) I/A] \\ 0 & \text{for} \quad t > t_0 + x(t_0) I/A \end{cases}$$

Since $X^{-i}(t)$ is decreasing, $\int_{t_0}^{T} a^i(t)\, X^{-i}(t)\, dt$ is minimized by choosing the control variable as stated in the proposition.■

A.2 Proof of Proposition 2

Suppose that $x(t_0) \geq \frac{I^p - 1}{I - 1}\bar{x}$. A distressed trader's strategy is optimal, given the other traders' actions, since (i) until time $t_0 + \tau$, the price is falling and the distressed trader is selling as fast as he can without incurring temporary impact costs; and (ii) after time $t_0 + \tau$, the price is rising and the distressed trader is selling at the minimal required speed.

To see the optimality of a predator's strategy, suppose, without loss of generality, that trader i is not the trader in financial crisis and that all other traders are using the proposed equilibrium strategies. Then, the total position, $X^{-i}(t)$, of the other traders is

$$X^{-i}(t) = \begin{cases} (I-1)\,(x(t_0) - \frac{A}{I}t) & \text{for} \quad t \in [t_0, t_0 + \tau] \\ (I^p - 1)\,\bar{x} & \text{for} \quad t > t_0 + \tau. \end{cases}$$

Trader i wants to minimize $\int_{t_0}^{T} a^i(t)\, X^{-i}(t)\, dt$ subject to the constraints that $x^i(T) = \bar{x}$ and $a^i(t) \in [\underline{a}, \overline{a}]$. Since $X^{-i}(t)$ is decreasing and then constant, $\int_{t_0}^{T} a^i(t)\, X^{-i}(t)\, dt$ is minimized by choosing $a^i = \underline{a}$ as long as $X^{-i}(t)$ is decreasing and by choosing a positive a^i thereafter. Hence, the a^i that is described in the proposition is optimal. We note that trader i's objective function does not depend on the speed with which he buys back after time τ. There is a single speed, however, which is consistent with the equilibrium.

To prove the uniqueness of this equilibrium, we note that, in any symmetric equilibrium, X^{-i} must satisfy the following: (i) $X^{-i}(t) > (I^p - 1)\,\bar{x}$ implies $\dot{X}^{-i}(t) = -A\frac{I-1}{I}$ (ii) $X^{-i}(t) < (I^p - 1)\,\bar{x}$ implies $\dot{X}^{-i}(t) = A\left(1 - \frac{I+I^c}{I^p I}\right) > 0$. It is easily seen that there is a unique $X^{-i}(t)$ satisfying these two conditions.■

A.3 Proposition 2' and Its Proof

Proposition 2' *Suppose that $x(t_0) < \frac{I^p-1}{I-1}\bar{x}$ and let $\tau := -\frac{x(t_0) - \frac{I^p-1}{I-1}\bar{x}}{A\left(1-\frac{I+I^c}{I^p I}\right)}$. The unique symmetric equilibrium strategy is for each predator $i \in I^p$ to buy fast for τ periods and keep buying at a lower trading intensity until $t_0 + \frac{x(t_0)}{A/I}$. More precisely,*

$$a^{i*}(t) = \begin{cases} \frac{A(I+I^p)}{I^p I} & \text{for} \quad t \in [t_0, t_0 + \tau) \\ \frac{AI^c}{I(I^p-1)} & \text{for} \quad t \in [t_0 + \tau, t_0 + \frac{x(t_0)}{A/I}) \\ 0 & \text{for} \quad t \geq t_0 + \frac{x(t_0)}{A/I}. \end{cases} \tag{17}$$

Proof
Analogous to the proof of Proposition 2.■

A.4 Proof of Proposition 3

The size of the overshooting, that is, the difference between the lowest price (which is achieved at time $t_0 + \tau$) and the new equilibrium price is $\bar{x}I^c/(I-1)$. This difference approaches zero as the number of agents increases, i.e.,

$$\frac{\bar{x}I^c}{I-1} = \frac{(\bar{x}I)\,(I^c/I)}{I-1} \rightarrow 0,$$

since $\bar{x}I$ and I^c/I are constant. ■

A.5 Proof of Proposition 4

To show that $\underline{W}(I^c)$ is increasing in I^c, we show that the paper wealth, $W^i(t, a^i, a^{-i})$, at any time t is (weakly) decreasing in I^c. The paper wealth is increasing in the holding, X^{-i}, of the other traders. With higher I^c, more agents are liquidating their entire holding, reducing X^{-i}. Further, the remaining predators reverse from selling to buying at a later time since τ (defined in Proposition 2) is increasing in I^c, which also reduces X^{-i}. ■

A.6 Proof of Proposition 5

Clearly, $V^{paper} > V^{orderly}$. If there is only one predator, then it follows immediately from Proposition 1 that $V^{orderly} > V^{distressed}$. If there are multiple predators, the distressed liquidation value is computed using Proposition 2. After tedious calculations, the result is

$$V^{distressed} = x(t_0)p(t_0) - \frac{1}{2}\lambda \left(Ix(t_0)^2 - \frac{I^p\left(I^p - 1\right)}{I - 1}\bar{x}^2 \right) . \tag{18}$$

It follows that $V^{orderly} > V^{distressed}$ under the condition stated in the proposition. ■

A.7 Proof of Proposition 6

We note first that $Ix(t_0) \geq \sqrt{I^p I}\bar{x}$ implies that, for all I, $x\left(t_0\right) \geq \frac{I^p-1}{I-1}\bar{x}$. Hence, Proposition 2 applies and we can use (18) to compute the total distressed liquidation value:

$$I^c V^{distressed} = I^c x(t_0)p(t_0) - \frac{1}{2}\lambda \left(I^c I x(t_0)^2 - \frac{\left(I^p - 1\right)}{I - 1} I^c I^p \bar{x}^2 \right) . \tag{19}$$

Since $Ix(t_0)$, $I^p x(t_0)$, and $I^c x(t_0)$ are assumed independent of I, all the terms in (19) are independent of I, except the term involving $(I^p - 1)/(I - 1)$. This term is increasing in the number of agents, yielding the first result in the proposition. In the limit as the number of agents increases, the total distressed liquidation value is

$$I^c x(t_0)p(t_0) - \frac{1}{2}\lambda \left(I^c I x(t_0)^2 - \frac{I^p}{I} I^c I^p \bar{x}^2 \right) . \tag{20}$$

This value is greater than the orderly liquidation value, $I^c x(t_0)\left(p(t_0) - \frac{1}{2}\lambda I^c x(t_0)\right)$, under the condition $Ix(t_0) \geq \sqrt{I^p I}\bar{x}$. ■

A.8 Proof of Proposition 7

We give a sketch of the proof. To see the optimality of trader i's strategy, we first note that, for any value of $x^i(t_0)$, it is optimal to use the equilibrium strategy after time t_0. The argument for this follows from the proofs of Propositions 1 and 2. Further, prior to t_0, it is optimal to acquire shares at a rate of $\overline{a}$ until the trader has reached his pre-t_0 target. This follows from the incentive to acquire the position before other traders drive up the price.

The equilibrium level of $x(t_0)$ is derived in the remainder of the proof. We consider trader i's expected profit in connection with buying $x(t_0) + \Delta$ shares, given that other traders buy $x(t_0)$ shares. More precisely, we use Lemma 1 and consider how the marginal Δ infinitesimal shares affect the "trading cost" $\int a^i(t) X^{-i}(t) dt$. First, buying Δ infinitesimal extra shares prior to time t_0 costs $\Delta(I-1)x(t_0)$ since the shares are optimally bought when the other traders have finished buying and $X^{-i} = (I-1)x(t_0)$.

The benefit, after t_0, of having bought the Δ shares depends on whether (i) no trader is in distress, (ii) another trader is in distress, or (iii) the trader himself is in distress:

(i) If no trader is in distress, then having the extra Δ shares saves a purchase at the per share cost of $X^{-i} = (I-1)\bar{x}$. This is because the marginal shares are bought in the end when the other $(I-1)$ traders each have acquired a position of $\bar{x}$.

(ii) If another trader is in financial distress then having the extra Δ shares saves a purchase at the per-share cost of $X^{-i} = (I-2)\bar{x}$. This is the total position of the other $I-2$ predators when the defaulting trader has liquidated his entire position.

(iii a) Suppose $I > 2$ and the position of the distressed trader is not disclosed at time t_0. Then, if the trader himself is in financial distress, the extra Δ shares can be sold when $X^{-i} = (I-1)\bar{x}$. This is the position of the predators when one has just finished liquidating. At that time the predators have preyed and re-purchased their position.

(iii b) Suppose $I = 2$ or that the position of the distressed trader is disclosed at time t_0. Then, if the trader himself is in financial distress, the extra Δ shares can be sold when $X^{-i} = (I-2)\bar{x}$. To see this, note that the extra shares imply that the predators prey longer (τ larger) because they know that one must liquidate a larger position. Hence, the marginal shares are effectively sold at the worst time when X^{-i} is at its lowest point.

We can now derive the equilibrium $x(t_0)$ by imposing the requirement that the marginal cost of buying the extra shares equals the marginal benefit. In the case in which $I > 2$ and the position of the distressed trader is not disclosed at time t_0, we have

$$(I-1)x(t_0) = (1-\pi)(I-1)\bar{x} + \pi \frac{I-1}{I}(I-2)\bar{x} + \pi \frac{1}{I}(I-1)\bar{x},$$

implying that

$$x(t_0) = (1 - \frac{\pi}{I})\bar{x}.$$

In the case in which $I = 2$ or the position of the distressed trader is disclosed at time t_0, we have

$$(I-1)x(t_0) = (1-\pi)(I-1)\bar{x} + \pi \frac{I-1}{I}(I-2)\bar{x} + \pi \frac{1}{I}(I-2)\bar{x},$$

implying that

$$x(t_0) = (1 - \frac{\pi}{I-1})\bar{x}.$$

The global optimality of buying $x(t_0)$ shares is seen as follows. First, buying fewer shares than $x(t_0)$ is not optimal since the infra-marginal shares are bought cheaper (in terms of X^{-i}) than $(I-1)\bar{x}$ and their expected benefits are at least $(I-1)\bar{x}$. Second, buying more shares than $x(t_0)$ costs $(I-1)\bar{x}$ per share, and the expected benefit of these additional shares is at most $(I-1)\bar{x}$. ■

A.9 Proof of Proposition 8

The proof of this proposition follows directly from the insight that, from each predator's view point, X^{-i} declines with a constant slope $\frac{I^p-1}{I^p}A$ and stays flat as soon as it reaches its final level $X^{-i}(T) = (I^p - 1)\overline{x}$. That front-running implies more costly liquidation and greater price overshooting follows from (i) that each predator has a smaller position at all times; (ii) that the lowest total holding of all agents (at time t_1) is strictly lower. ■

A.10 Proof of Proposition 9

The execution price for trader i's order of u^i shares in the batch auction is $p^a(u^i) = \mu - \lambda S + \lambda(I^p - 1)(x(t_0) + u^{-i}) + \lambda(x(t_0) + u^i)$ if all defaulting traders submit sell orders of size $x(t_0)$ and all other predators submit a total buy order of $u^{-i} = (I^p - 1)u$, where u is the equilibrium predator buy order in the auction (to be determined). Since X^{-i} is increasing after the auction, trader i optimally buys the remaining $[\overline{x} - x(t_0) - u^i]$ shares at the highest trading intensity A/I^p. If $u^i \geq u$, these buy orders are executed at an average price of $p^a + \frac{1}{2}\lambda I^p[\overline{x} - x(t_0) - u^i]$. Deriving the total cost and taking the first order condition w.r.t. u^i yields an optimal auction order of $u^i = \frac{I^p-1}{I^p}[\overline{x} - x(t_0)]$. If $u^i < u$, then predator i must buy more shares after the auction than the other predators. The first u shares are bought at an average price of $p^a + \frac{1}{2}\lambda I^p[\overline{x} - x(t_0) - u]$ and the last $u - u^i$ ones are bought at an average price of $p^a + \lambda I^p[\overline{x} - x(t_0) - u] + \frac{1}{2}\lambda(u - u^i)$. Taking the first order condition w.r.t. u^i evaluated at $u^i = u$, we find the same optimality condition as above: $u^i = u = \frac{I^p-1}{I^p}[\overline{x} - x(t_0)]$.

The equilibrium auction price is $p^a = \mu - \lambda S + \lambda(I^p - 1)\overline{x} + \lambda x(t_0)$. The price overshooting is $\lambda(\bar{x} - x(t_0))$, which is smaller than the price overshooting without batch auction, $\lambda\frac{I^c}{I-1}\bar{x}$ for $I^p \geq 2$ and $\lambda\overline{x}$ for $I^p = 1$ (Propositions 1 and 2, respectively). ■

B Noisy Asset Supply

In this section, we consider an economy in which (i) agents can condition on prices and (ii) the supply of assets is noisy. In a pure strategy equilibrium, an agent cannot learn from prices if another trader deviated from his equilibrium strategy. This is because, in equilibrium, any price change is ascribed to supply uncertainty. This feature of the noisy supply equilibrium is the justification for the equilibrium concept described in Definition 1. We show that the equilibrium strategies for the non-noisy economy also constitutes an equilibrium in a corresponding game with noisy supply.

We assume that the supply, S_t, is a Brownian motion with volatility σ, that is,

$$dS_t = \sigma dW_t,$$

where W is a standard Brownian motion. Agent i maximizes his expected wealth

$$\max_{a(\cdot)\in\mathcal{A}} E\left(-\int_0^T a^i(t)p(t)dt + x^i(T)v\right), \tag{21}$$

where A is the set of $\{\mathcal{F}_t\}$-adapted processes and $\{\mathcal{F}_t\}$ is generated by the price process $\{p_t\}$, the crisis indicator $\iota 1_{(t\geq t_0)}$ and by $G\left(a^i(t), a^{-i}(t)\right)$. The price is defined as before by $p\left(t\right) = \mu - \lambda\left(S_t - X\left(t\right)\right)$, where $S_0 > I$. We use the definition $\bar{p}\left(t\right) = \mu - \lambda\left(S_0 - X\left(t\right)\right)$. With this definition, the agent's objective function can be written as

$$\begin{aligned} E\left(x^i(T)v - \int_0^T [a^i(t)p(t) + G\left(a^i(t), a^{-i}(t)\right)]dt+\right) = \\ E\left(x^i(T)v - \int_0^T [a^i(t)\bar{p}(t) + G\left(a^i(t), a^{-i}(t)\right)]dt\right) + E\int_0^T a^i(t)\left[\bar{p}(t) - p(t)\right]dt. \end{aligned}$$

The first term on the right-hand side is the same as the objective function with a constant supply of S_0. Hence, this term is maximized by the equilibrium strategy if all other agents use the equilibrium strategy. The second term is, as we show next, zero under an additional assumption. For any $\{\mathcal{F}_t\}$-adapted process, a, it holds that

$$\begin{aligned} & E\int_0^T a^i(t)\left[\bar{p}(t) - p(t)\right]dt \\ & = \lambda E\int_0^T a^i(t)\left[S_0 - S_t\right]dt \\ & = \lambda E\int_0^T a^i(t)\left[S_T - S_t\right]dt - \lambda E\left(\int_0^T a^i(t)dt\left[S_T - S_0\right]\right) \\ & = \lambda E\int_0^T a^i(t)E_t\left[S_T - S_t\right]dt - \lambda E\left(\int_0^T a^i(t)dt\left[S_T - S_0\right]\right) \\ & = -\lambda E\left(\int_0^T a^i(t)dt\left[S_T - S_0\right]\right). \end{aligned}$$

If we assume that the agent must choose a strategy with $\bar{x} = x_T = x_0 + \int_0^T a^i(t)dt$, then the last term is zero. This assumption means that the agent must end up fully invested in the asset. We note that the agent would optimally choose $x_T = 1$ as long as S is not too small, and S is close to S_0 with large probability if σ is small.

Even if we do not impose the additional assumption that $x_T = \bar{x}$, we can show that the equilibrium in the model without supply uncertainty is an ε-equilibrium in the model with noisy supply. (See Radner (1980) for a discussion of ε-equilibria.) This property of the strategies follows from the fact that the latter term can be bounded as follows

$$\begin{aligned}
\left| E\left(\int_0^T a^i(t)dt\,[S_T - S_0]\right)\right| &\leq E\left(\int_0^T \left|a^i(t)\right| dt\ \ |S_T - S_0|\right) \\
&\leq E\left(\int_0^T \left|a^i(t)\right| dt\ \ |S_T - S_0|\right) \\
&\leq E\left(\int_0^T cdt\ \ |S_T - S_0|\right) \\
&\leq cT\ E\left|S_T - S_0\right| \\
&= cT\ \sigma E\left|W_T - W_0\right| \\
&\rightarrow 0 \qquad \text{as } \sigma \rightarrow 0.
\end{aligned}$$

Hence, agent i's maximal gain from deviating from the strategy of the non-noisy game approaches 0 as the supply uncertainty vanishes ($\sigma \rightarrow 0$).

Table 1: Examples of risks associated with predatory trading.

Issue	Source	Quotation
Enron, UBS Warburg	AFX News Limited, AFX - Asia, January 18, 2002.	UBS Warburg's proposal to take over Eron's traders without taking over the trading book was opposed on the ground that "it would present a 'predatory trading risk', as Enron traders effectively know the contents of the trading book."
Disclosure	IAFE Investor Risk Committee (IRC) (2001)	"For large portfolios, granular disclosure is far from costless and can be ruinous. Large funds need to limit granularity of reporting sufficiently to protect against predatory trading."
Predation, LTCM	Business Week, 2/26/01	"If lenders know that a hedge fund needs to sell something quickly, they will sell the same asset – driving the price down even faster. Goldman Sachs and counterparties to LTCM did exactly that in 1998. (Goldman admits it was a seller but says it acted honorably and had no confidential information.)"
LTCM	New York Times Magazine, 1/24/99	Meriwether quoting another LTCM principal: "the hurricane is not more or less likely to hit because more hurricane insurance has been written. In financial markets this is not true ... because the people who know you have sold the insurance can make it happen."
Systemic risk	Testimony of Alan Greenspan, U.S. House of Representatives, 10/1/98	"It was the judgment of officials at the Federal Reserve Bank of New York, who were monitoring the situation on an ongoing basis, that the act of unwinding LTCM's portfolio in a forced liquidation would not only have a significant distorting impact on market prices but also in the process could produce large losses, or worse, for a number of creditors and counterparties, and for other market participants who were not directly involved with LTCM."
Askin/ Granite Hedge Fund collapse 1994	Friedman Kaplan Seiler & Adelman LLP	"[during] the period around which the rumors as to the Funds' difficulties were circulating, DLJ quickly repriced the securities resulting in significant margin deficits. ... The court also cited evidence that Merrill may have improperly diverted profits to itself."
Predation	Securities Week, 11/11/91	"knowledge that other customers intend to sell large amounts of stock should a specified decline occur in the stock's price... the dishonest customer's stock selling will trigger ... selling done by other customers ... result in a large decline in the stock's price."
Market making	Financial Times (London), 6/5/1990, section I, page 12.	UK market makers wanted to keep the right to delay reporting of large transactions since this would give them "a chance to reduce a large exposure, rather than alerting the rest of the market and exposing them to predatory trading tactics from others."
Crash 1987	Brady Report	"This precipitous decline began with several "triggers," which ignited mechanical, price-insensitive selling by a number of institution following portfolio insurance strategies and a small number of mutual fund groups. The selling by these investors, and the prospect of further selling by them, encouraged a number of aggressive trading-oriented institutions to sell in anticipation of further declines. These aggressive trading-oriented institutions included, in addition to hedge funds, a small number of pension and endowment funds, money management firms and investment banking houses. This selling in turn stimulated further reactive selling by portfolio insurers and mutual funds."

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